Vector-Meson Fit to Nucleon Form Factors*

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(Received 31 May 1963)

Available data on nucleon electric and magnetic form factors have been fitted with a dispersion theoretic formula, assuming the electron-nucleon scattering process dominated by the ρ, ω, and ϕ mesons and hard cores. The data are found to fit a smooth curve of the assumed form. The fit, based mainly on proton data, predicts neutron form factors compatible with experimental results.

WE are attempting to fit known proton electric and magnetic form factors $G_{Ep}(q^2)$ and $G_{Mp}(q^2)$ by means of a dispersion relation

$$G(-q^2)\equiv G(t)=\frac{1}{\pi}\int_{4\mu^2}^{\infty}\frac{g(t')dt'}{t'-t}+G(-\infty)\,, \qquad (1)$$

where $-t=q^2\geqslant 0$ is the invariant four-momentum transfer and μ is the mass of the pion. Certain trial forms of the spectral function $g(t')$ are selected and the resulting $G(q^2)$ compared with the experimental data.

Previous attempts of this nature have restricted themselves to 2-pole fits, in which $g(t')$ is approximated by a sum of two delta functions, of variable strengths, at certain t' values.[1] This approach has had considerable success in fitting the proton form factors. We report here on a fit achieved by assuming that the electron-nucleon scattering process is dominated by certain resonant intermediate states, namely, the vector mesons ρ, ω, and ϕ. The masses, widths, and quantum numbers (1^-) of these states have been experimentally determined,[2] but the strengths with which they enter into the reaction are unknown, and serve as the variable parameters in our fit. All higher mass intermediate states are represented by the subtraction constants (sometimes referred to as "hard cores") $G(-\infty)$.

Electron-proton scattering data are now available for a fairly wide range of values of q^2 $(0\leq q^2\leq 45\text{ F}^{-2})$ (see Table I). These data have been analyzed in terms of proton form factors, utilizing the one-photon-exchange model and the resultant Rosenbluth formula.[3] These form factors, together with their errors, form the raw data for our fit. It should be noted that G_{Mp} is closely determined over the entire experimental range, while G_{Ep} is less well determined at high-momentum transfers. (The recent, accurate Cambridge Electron Accellerator data in this range were not used in the fit, but are shown on the curves.) We have also made use of the values of G_{Mn} obtained from electron-neutron coincidence work by the Cornell group, at momentum transfers of 4.9 F^{-2} and 11.0 F^{-2} (see Table I). The G_{En} values are available as a check on the fit, but were not used in the calculations. (We should point out here that the proton data determine accurately only the sum of the contributions from the closely spaced ρ and ω resonances, and that some neutron data are required to effect a separation of these two terms.)

Certain very accurately known data have been used as constraints on the fit. The static values $G_{Ep}(0)=1.0$, $G_{En}(0)=0.0$, $G_{Mp}(0)=2.793$, and $G_{Mn}(0)=-1.913$ and also the slope of G_{En} at the origin[4] $(dG_{En}/dq^2)_{q^2=0}=0.021\pm0.001\text{ F}^2$ are sufficiently well known to be used as constraints on the formulas, rather than as single data points in the fit.

We represent the ρ meson by a Breit-Wigner formula,

$$g_\rho(t')=A_\rho\left[\frac{(\gamma/2)t_\rho}{(t'-t_\rho)^2+\gamma^2/4}\right],\ t'\geqslant 8\mu^2\,, \qquad (2)$$

where $t_\rho=m_\rho{}^2=14.6\text{ F}^{-2}$ and $\gamma=2m_\rho\Gamma$, $\Gamma=100$ MeV. A_ρ is a normalization constant. The correct P-wave threshold behavior has been ensured by utilizing

$$g_\rho{}^0(t')=C(t'-4\mu^2)^{3/2}\quad\text{for}\quad 4\mu^2\leq t'\leq 8\mu^2. \qquad (3)$$

C is chosen so that $g_\rho{}^0(8\mu^2)=g_\rho(8\mu^2)$. The function $G_\rho(q^2)$ obtained by substituting $g_\rho(t')$ into the dispersion relation and evaluating the integral [with $G(-\infty)=0$] is normalized to unity at $q^2=0$. This function approximates closely a pole at 13.0 F^{-2}. *Note added in proof.* Wong[5] has shown that the apparent positions of the resonances may be shifted from their experimental values when used in fitting form factors. A part of his shift corresponds to this "width effect."

The extremely small experimental widths of the ω and ϕ mesons makes it reasonable to assume delta-function forms for their spectral functions,

$$\begin{aligned} g_\omega(t')&=\pi t_\omega\delta(t'-t_\omega),\\ g_\phi(t')&=\pi t_\phi\delta(t'-t_\phi), \end{aligned} \qquad (4)$$

with $t_\omega=m_\omega{}^2=16.0\text{ F}^{-2}$ and $t_\phi=m_\phi{}^2\doteq 26.7\text{ F}^{-2}$. The multiplying factors are chosen such that the resulting $G_\omega(q^2)$ and $G_\phi(q^2)$ are normalized to unity at the origin.

* Supported in part by the U. S. Office of Naval Research.

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TABLE I. Relevant Data.

		Proton			
q^2 (F^{-2})	G_{Ep}	ΔG_{Ep}	G_{Mp}	ΔG_{Mp}	Refs.
0.30	0.970	0.004			c, f
0.49	0.932	0.009			f
0.60	0.940	0.006			c
1.00	0.885	0.005	2.508	0.038	c, f
1.05	0.884	0.009			e
1.60	0.850	0.010	2.394	0.025	c
2.00	0.784	0.012	2.232	0.034	f
2.20	0.790	0.006			c
2.98	0.725	0.022	2.035	0.016	e
4.56	0.629	0.054	1.620	0.110	a
7.0	0.522	0.119	1.390	0.343	a
9.0	0.475	0.033	1.130	0.075	a
10.0	0.417	0.02	1.119	0.045	d
11.5	0.333	0.045	1.026	0.052	a
13.0	0.300	0.060	0.931	0.056	a
15.0	0.290	0.052	0.855	0.050	a
16.5	0.323	0.035	0.727	0.023	a
18.0	0.347	0.034	0.637	0.032	a
21.5	0.305	0.045	0.544	0.019	a
25.0	0.362	0.034	0.469	0.011	a, b
30.0	0.359	0.037	0.382	0.014	b
35.0	0.258	0.044	0.314	0.012	b
40.0	0.436	0.073	0.232	0.018	b
45.0	0.0	0.255	0.238	0.022	b
		Neutron			
q^2 (F^{-2})	G_{En}	ΔG_{En}	G_{Mn}	ΔG_{Mn}	Refs.
4.9	0.16	0.04	−1.06	0.13	g
11.0	0.27	0.05	−0.79	0.09	h
		Recent proton data (not used in fit)			
q^2 (F^{-2})	G_{Ep}	ΔG_{Ep}	G_{Mp}	ΔG_{Mp}	Refs.
6.0	0.526	0.021			i
10.0	0.423	0.022			i
14.0	0.365	0.027			i
18.0	0.31	0.026			i
4.0	0.696	0.032	1.50	0.29	j
10.0	0.396	0.047	1.13	0.11	j

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This form of the spectral function gives rise to a Clementel-Villi form for G.

We write

$$G_{Ep}=G_{ES}+G_{EV},\quad G_{En}=G_{ES}-G_{EV},$$
$$G_{Mp}=G_{MS}+G_{MV},\quad G_{Mn}=G_{MS}-G_{MV},\tag{5}$$

where S and V refer to isoscalar and isovector parts, respectively, and these equations serve to define the right-hand sides. We know that ρ is an isovector particle, ω and ϕ isoscalar particles, so we may write

$$G_{EV}=\alpha_\rho G_\rho+\alpha_{EV};\quad G_{ES}=\alpha_\omega G_\omega+\alpha_\phi G_\phi+\alpha_{ES};$$
$$G_{MV}=\beta_\rho G_\rho+\beta_{MV};\quad G_{MS}=\beta_\omega G_\omega+\beta_\phi G_\phi+\beta_{MS}.\tag{6}$$

From the known static values of the G's, we conclude that the cores $G(-\infty)$ are given by

$$\alpha_{EV}=0.5-\alpha_\rho,\qquad \alpha_{ES}=0.5-\alpha_\omega-\alpha_\phi,$$
$$\beta_{MV}=2.353-\beta_\rho,\quad\text{and}\quad \beta_{MS}=0.440-\beta_\omega-\beta_\phi.\tag{7}$$

In addition, the known value of $(dG_{En}/dq^2)_{q^2=0}$ gives rise to the relation

$$\alpha_\phi=2.051\alpha_\rho-1.667\alpha_\omega-0.560.\tag{8}$$

We use 24 experimental values of G_{Ep} (shown in the table) in fitting

$$(G_{Ep}+0.560G_\phi-1.560)=\alpha_\rho(G_\rho+2.051G_\phi-3.051)+\alpha_\omega(G_\omega-1.667G_\phi+0.667),\tag{9}$$

and 19 experimental values of G_{Mp} and two experimental values of G_{Mn} (also in the table) in fitting

$$(G_M-K)=\tau\beta_\rho(G_\rho-1)+\beta_\omega(G_\omega-1)+\beta_\phi(G_\phi-1),\tag{10}$$

where $K_p=2.793$, $K_n=-1.913$, $\tau_p=+1$, $\tau_n=-1$. Equations (9) and (10) have been obtained by combining Eqs. (5) through (8). Thus, we have 22 degrees of freedom in fitting the electric form factor and 18 degrees of freedom in fitting the magnetic form factor. A least-squares analysis was performed, yielding the best values of α_ρ, α_ω, β_ρ, β_ω, and β_ϕ, their errors and correlated errors, and the χ^2 for the fits. α_ϕ and the subtraction constants $\frac{1}{2}(\alpha_{ES}\pm\alpha_{EV})$ and $\frac{1}{2}(\beta_{MS}\pm\beta_{MV})$ were then calculated, together with their errors.

We obtained $\chi_E^2=30.6$ for the electric form-factor fit and $\chi_M^2=22.1$ for the magnetic form-factor fit. The former could be exceeded by chance in a set of measurements on points known to fit the formula used in 10% of cases, the latter in 25% of cases. Hence, we conclude that we are able to make a good fit to the data in both cases. Our fits give

$$G_{Ep}=(0.898\pm0.014)G_\rho+(2.636\pm0.150)G_\omega+(-3.112\pm0.225)G_\phi+(0.578\pm0.065),\tag{11}$$

with the correlation coefficient between α_ρ and α_ω equal to $r_{\rho\omega}=0.883$;

$$G_{Mp}=(3.174\pm0.092)G_\rho+(3.448\pm0.243)G_\omega+(-3.825\pm0.262)G_\phi+(-0.004\pm0.051),$$

with $r_{\rho\omega}=-0.498$, $r_{\rho\phi}=0.112$, and $r_{\omega\phi}=-0.913$;

$$G_{En}=(-0.898\pm0.014)G_\rho+(2.636\pm0.150)G_\omega+(-3.112\pm0.225)G_\phi+(1.374\pm0.087),$$
$$G_{Mn}=(-3.174\pm0.092)G_\rho+(3.448\pm0.243)G_\omega+(-3.825\pm0.262)G_\phi+(1.638\pm0.190).\tag{12}$$

The assumption that electron-nucleon scattering is dominated by these three vector-meson intermediate states and hard cores thus gives a good fit to the experimental G_{Ep} and G_{Mp} data (see Fig. 1), and predicts G_{En} and G_{Mn} behavior consistent with the Cornell electron-neutron coincidence measurements of those quantities. The four Cambridge Electron Accelerator points fit the G_{Ep} curve with a χ^2 of about 6 which is entirely acceptable. The predicted G_{En} and G_{Mn} (see Fig. 2) appear to be consistent with other experimental data on the electric and magnetic form factors of the neutron,[6] though the experimental values are rather poorly determined, considering the inaccuracies arising in the analysis of the measured cross sections. Even aside from the validity of the theoretical model, our results indicate that the present nucleon form factors do lie on a smooth curve, and are thus mutually consistent in this sense. Equation (11) is very useful, at least, for purposes of interpolation.

[*Note added in proof*. If proton data alone are considered, the closeness of the ρ and ω reduces the fit to a two-pole fit, utilizing a "resonance" ($\rho-\omega$) and a "soft core" (ϕ). Extensive statistical analysis has shown[7] that the goodness of the fit is extremely insensitive to the positions of these poles, fits as good as, or even better than, the one reported here being obtained over a wide range of positions. Our approach would thus not necessarily be invalidated by changes in the number and positions of experimentally determined resonances.]

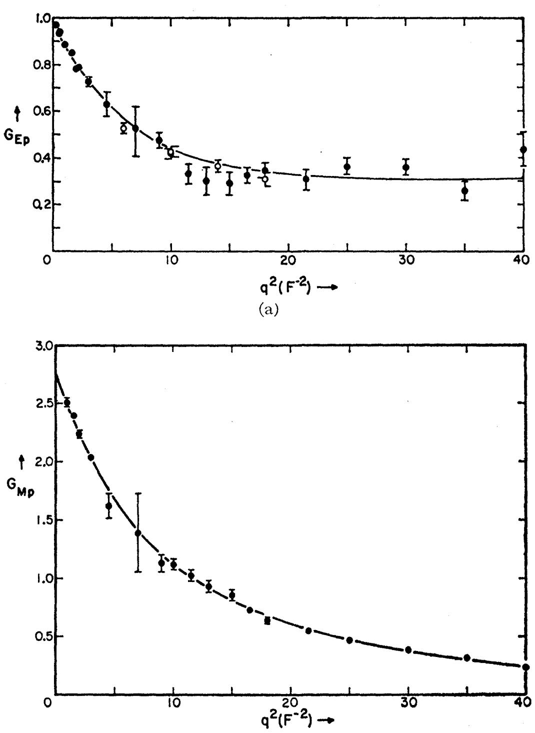

FIG. 1. Fits to proton form factors. (a) shows the experimental points and vector meson fit for the electric form factor G_{Ep}, (b) the experimental points and vector meson fit for the magnetic form factor G_{Mp}. The experimental points are taken from Table I (the open circles represent Cambridge Electron Accelerator data, which were not used in the fit) and the curves from Eq. (11).

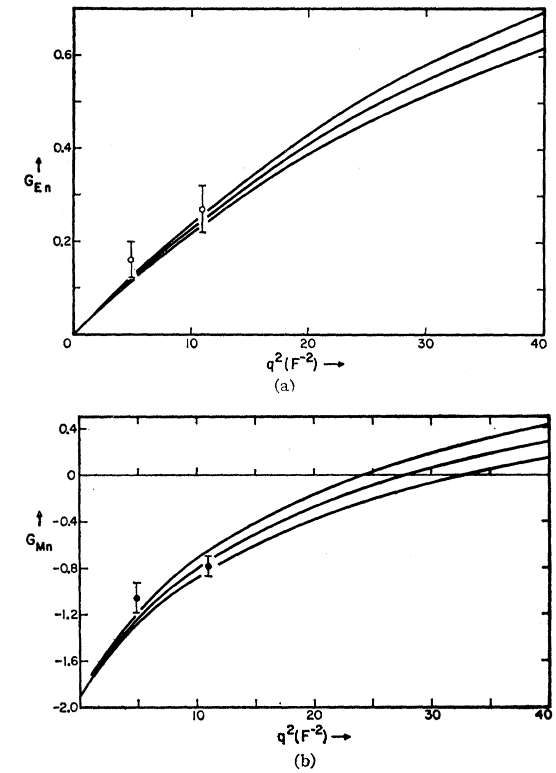

FIG. 2. Predicted neutron form factors. (a) shows the predicted values of G_{En}, together with the limits imposed by the statistical errors in the parameters and their correlations; (b) the predicted values of G_{Mn}, with similar limits. The points shown on the curves are the Cornell electron-neutron coincidence measurement values, which were used only in the G_{Mn} fit. The curves are from Eq. (12).

Our fits predict rather large positive neutron cores (it should be noted that G_ρ, G_ω, and G_ϕ all approach zero in the high-q^2 limit), and smaller or vanishing proton cores. We can calculate the isoscalar and isovector cores as

$$\begin{aligned}\alpha_{ES}&=0.976\pm0.076; \quad \alpha_{EV}=-0.398\pm0.014;\\ \beta_{MS}&=0.817\pm0.105; \quad \beta_{MV}=-0.821\pm0.092.\end{aligned} \tag{13}$$

I am indebted to J. S. Levinger and R. F. Peierls of this Laboratory for discussions and suggestions in connection with this analysis, and to P. Stein and K. Berkelman, also of this Laboratory, and R. Wilson of Harvard for as yet unpublished experimental data.

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